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# PARAMETER DESIGN AND OPTIMAL CONTROL OF AN OPEN CORE FLYWHEEL ENERGY STORAGE SYSTEM\*

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## ABSTRACT

In low earth orbit [LEO] satellite applications spacecraft power is provided by photovoltaic cells and batteries. To overcome battery shortcomings the University of Maryland, working in cooperation with NASA/GSFC and NASA/LeRC, has developed a magnetically suspended flywheel for energy storage applications. The system is shown in Figure 1 and is referred to as an Open Core Composite Flywheel [OCCF] energy storage system.

Successful application of flywheel energy storage requires integration of several technologies, viz. bearings, rotor design, motor/generator, power conditioning, and system control.

In this paper we present a parameter design method which has been developed for analyzing the linear SISO model of the magnetic bearing controller for the OCCF shown in Figure 2. The objective of this continued research is to principally analyze the magnetic bearing system for nonlinear effects in order to increase the region of stability, as determined by high speed and large air gap control. This is achieved by four tasks: (1) physical modeling, design, prototyping, and testing of an improved magnetically suspended flywheel energy storage system, (2) identification of problems that limit performance and their corresponding solutions, (3) development of a design methodology for magnetic bearings, and (4) design of an optimal controller for future high speed applications.

Both nonlinear SISO and MIMO models of the magnetic system were built to study limit cycle oscillations and power amplifier saturation phenomenon observed in experiments. The nonlinear models

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include the inductance of EM coils, the power amplifier saturation, and the physical limitation of the flywheel movement as discussed earlier. The control program EASY5 is used to study the nonlinear SISO and MIMO models.

Our results have shown that the characteristics and frequency responses of the magnetic bearing system obtained from modeling are comparable to those obtained experimentally. Although magnetic saturation is shown in the bearings, there are good correlations between the theoretical model and experimental data. Both simulation and experiment confirm large variations of the magnetic bearing characteristics due to air gap growth. Therefore, the gap growth effect should be considered in the magnetic bearing system design.

Additionally, the magnetic bearing control system will be compared to other design methods using not only parameter design but  $H^\infty$  optimal control and  $\mu$  synthesis.

## INTRODUCTION

Although the control system and the magnetic bearing are an integral system, it is possible to independently evaluate and optimize the performance of the control system for a given plant.

The design of the flywheel energy storage (FES) plant is based on a pancake-shaped PM/EM magnetic bearing and a spokeless composite flywheel proposed earlier [1,2]. Based upon this design, four different prototypes of the FES system have been built and tested in the Magnetic Bearing Laboratory. The first prototype was built using a 3-inch magnetic bearing, a DC brushless motor, an aluminum flywheel and the prototype tested at a maximum speed of 9,333 RPM to demonstrate system feasibility. Next a 3-inch magnetic bearing stack using two magnetic bearings without a motor and an aluminum flywheel was spun by an air jet up to 7,000 RPM. Finally, a prototype consisting of two 4-inch magnetic bearings, a commercial off-the-shelf PM DC brushless motor, and an aluminum flywheel was built. The 4-inch magnetically suspended flywheel energy storage system was tested with a very limited performance. Anand, et al. [3] developed a new FES system using two newly improved magnetic bearings, a high efficiency PM DC motor/generator, and an aluminum flywheel. The system achieved a maximum speed of 6,800 RPM in air. With improvements to the magnetic bearings, the motor/generator, the power/control electronics, the vacuum chamber, and the composite flywheel, Wells, Pang, and Kirk [4] tested the final FES system shown in Fig. 1 at a maximum speed of 20,000 RPM with a total stored energy of 15.9 WH and an angular momentum of 54.8 N-m-s (40.4 lb-ft-s).

This research is specifically concerned with control system performance for the bearing shown in Fig. 1 and discussed in reference [5]. It must be noted that the results obtained in this study apply over a broad range of magnetic bearing parameters and can therefore be thought of as general conclusions. The magnetic bearings and Flywheel Energy Storage System shown in Fig. 1 has been extensively studied from a design viewpoint and reported in [5]. The control of this bearing was achieved by the proportional and derivative (PD) control system whose block diagram is shown in Fig. 2. This SISO model is duplicated to provide four independent and identical PD control systems used to control four degrees of freedom in the FES system of Fig. 1

There have been many advancements in the control system design and dynamic analysis for the FES system. For the original linear single-input-single-output (SISO) control system for the magnetic bearing,

Jayaraman [6] derived dynamic equations and analyzed dynamic responses of a linear multi-input-multi-output (MIMO) model. Zmood, et al., [7] developed a nonlinear SISO model of magnetic bearing to study limit cycle oscillations. Anand, et al., [3] used a parameter optimization method and the JEYCAD program [8] to create a control system with desired gain and phase margins. Fittro [9] developed a hybrid multi-layered neural network controller for a nonlinear SISO magnetic bearing models. Wells, Pang, and Kirk [4] built an adjustable stiffness and damping controller for the final FES system.

The design of the control system is constrained by the characteristics of the magnetic bearing such as load capability and linear range. Figure 3 shows typical experimental testing results of the magnetic bearing. Curves A and C are plotted as force and control current responses versus displacement at a fixed gain of the control system. Curves B and D are plotted at a higher gain. It shows that the net restoring force reaches its peak when the power amplifier becomes saturated and cannot supply more current. The distance between the center and the peak is called the linear range where the bearing should be operated. The slopes of curves A and B are the radial stiffness of the bearing. Notice that the linear range is dependent on the gain of the control system, the maximum control current of the power amplifier, and the magnetic saturation of the material. The bearings should be designed to have a maximum linear range with a proper load carrying capability.

Two design methods, the component design and the parameter design, have been developed for analyzing the linear SISO model of Fig. 2. The parameter design adjusts the critical parameter values to satisfy the gain margin and phase margin specifications for a robust control system. The parameters can be derived by the selection of the resistance and/or capacitance in the electrical circuit. The JEYCAD software program [8] was specifically written to provide a component design tool for this application. It allows the user to input the component data of the control system, and it will compute the appropriate transfer functions and the bearing characteristics. The JEYCAD program includes the Classic Control (CC) software program, which allows the designer to plot the frequency and time response of the control system and compare it to the performance specifications.

Both nonlinear SISO and MIMO models of the magnetic system are built to study limit cycle oscillations and power amplifier saturation phenomenon observed in experiments. The nonlinear models include the inductance of EM coils, the power amplifier saturation, and the physical limitation of the flywheel movement as discussed earlier. The control program EASY5 is used to study the nonlinear SISO and MIMO models.

Simulation and experimentation have identified that the inductance of the EM coils and the voltage limitation of the power amplifier are the causes for magnetic suspension failure. After these problems were corrected, the FES system achieved a maximum speed of 20,000 RPM and was stopped by current limitation of the motor controller. At 20,000 RPM, there is no gap growth effect and the gyroscopic motion does not yet cause any instability. At this speed, the existing control system design based on the linear SISO model has been proved to be robust and reliable.

## MODELING AND VALIDATION

Physical modeling is critical to the success of the design and control for a magnetically suspended flywheel energy storage system. At low speeds, the system can be simplified and decoupled as a linear single-input-single-output (SISO) model. However, the linear model cannot explain some observed

phenomena such as the power amplifier saturation, the third harmonic noise, and the limit cycle oscillation. At high speeds, rotational stress causes flywheel deformation and air gap growth, which changes bearing characteristics. In addition, the gyroscopic effect of the flywheel couples the two radial axes motions so the system becomes a nonlinear multi-input-multiple-output (MIMO) problem. Detailed modeling is discussed in [5]. These models are refinements of earlier work and, in general, include nonlinear effects and disturbances.

Experimental tests have been conducted on the magnetic bearings and the FES system to validate the modeling of (1) the magnetic bearing characteristics, (2) the gap growth effects, and (3) the frequency responses of the magnetic bearing system.

The experimental results of the bearing characteristics are used to validate the mathematical model of the magnetic bearing used here [5]. The theoretical values of passive radial stiffness and axial stiffness are computed with the empirical data from flux density measurements. The active stiffness, maximum radial force, linear range and stable range are affected by maximum control current and the current displacement ratio of the feedback control.

There are reasonably good correlations between the theoretical and experimental data for stiffness and the linear range. The maximum radial force has a large modeling error due to magnetic saturation. It is apparent that the increase of the radial force slows as the displacement increases from the center. Another indication is that the slope of the corrective force decreases as the current increases. However, the maximum radial force is not critical since the magnetic bearing mostly operates within the range of  $\pm 0.038$  mm (0.0015 in) where the modeling error for the radial force is small.

Because the rotational stress causes an outward deformation of the flywheel and increases the air gap at high rotating speeds, the characteristics of magnetic bearing will change as a function of the speed. It has been estimated that the air gap is increased between 0.10 and 0.41 mm (0.004 and 0.016 in) when a composite flywheel rotates between 40,000 and 80,000 RPM. The increase is more than 40% of the nominal air gap of 1.02 mm (0.04 in) at zero speed. The large variation of the air gap will change bearing characteristics and affect the FES system performance. This phenomenon, called gap growth effect, cannot be ignored and must be considered in the magnetic bearing system design.

The mathematical equations for the gap growth effects suggest that the air gap growth has great impact on the change of bearing characteristics at high rotational speeds. In order to simulate the gap growth effect on the magnetic bearing system, all the radial dimensions of the return rings are enlarged by the same magnitude as the gap growth. Simulation shows that as the air gaps increase by 10% and 21% of the original value, the passive stiffness decreases to 22% and 37%, and the force current stiffness to 14% and 28% of their original values. Because the change of the bearing characteristics is much larger than the change of the air gap, the gap growth effect cannot be ignored in the high speed applications. With reasonable confidence in the plant model briefly discussed above, and detailed in [5], it is appropriate that the control system performance be further analyzed and optimized.

The frequency response of the FES system is analyzed using control software EASY5. The simulation has the following assumptions:

- (1) All the characteristics of the magnetic bearings are identical,
- (2) All the control systems and displacement sensors are the same,

- (3) The rotor dynamics of the system corresponds to a rigid body motion,
- (4) There is no electrical saturation or time delay in the power amplifier,
- (5) There is no magnetic saturation or nonlinearity in the magnetic bearings,
- (6) The resistance of the EM coils can be ignored,
- (7) The motor/generator has no effects on the magnetic bearing system.

The parameters of the control system, displacement sensor, and power amplifier for one-axis of the FES system are discussed in [5].

Figure 4 shows the displacement frequency responses of the FES system and its theoretical model. Both experimental and simulative results show similar trends with little difference. The FES system has a flat displacement frequency response until it reaches its natural frequency. The current frequency response of the system is almost constant throughout the frequency range. The voltage output always increases as the frequency increases but the trend in the actual system slows beyond 110 Hz. Similar conclusions can be reached from the current and frequency responses.

Validation of the characteristics and frequency responses of the magnetic bearing system indicate that there are good correlations between the theoretical model and experimental data.

## PARAMETER DESIGN

The existing controller for the magnetic bearing system was designed with a parameter design method that achieves desirable gain margin and phase margin discussed in the previous section. The existing controller provides a stable system operation up to 20,000 RPM. However, the magnetic bearing system has air gap growth at high rotating speeds which causes bearing characteristics to change. In this section, an optimal control system is proposed using  $H^\infty$  method and  $\mu$  synthesis to account for gap growth effect and other plant uncertainties at high rotational speeds. Although the existing controller can achieve stable performance at the maximum gap growth, it is still desired to have better performance with larger bandwidth and faster settling time.

A parameter design method, proposed by Chang and Han [10], is used to find the desired gain margin and phase margin of control systems with adjustable parameters. Consider a system having an open loop transfer function  $G(s)$  and a unit feedback loop with the system having adjustable parameters. The characteristic equation of the control system can be written in the frequency domain and expressed in terms of a real part and an imaginary part, with both parts equal to zero.

$$F_r(j\omega) = F_r(\alpha, \beta, \gamma, \dots, A, \theta, j\omega) = 0 \quad (1)$$

$$F_i(j\omega) = F_i(\alpha, \beta, \gamma, \dots, A, \theta, j\omega) = 0 \quad (2)$$

Any two parameters can be solved from these equations by keeping the rest of the parameters constant.

In order to achieve the desired gain margin and phase margin of the control system, the following three loci are drawn on the two-parameter plane.

- (1) The locus of  $A = 1$  and  $\theta = 0$
- (2) The locus of  $A = \text{gain margin}$  and  $\theta = 0$ ,
- (3) The locus of  $A = 1$  and  $\theta = \text{phase margin}$ .

The first locus is a boundary of marginal stability of the control system. The second locus is a boundary of a constant gain margin and the third locus is a boundary of a constant phase margin. The enclosed region of these three loci will satisfy the minimum gain margin and phase margin. After the parameters are chosen, the parameters can be written as the form of circuitry components. The component values can be found by solving a set of linear equations.

Since the gain and time constant of the zero in the control system are the two most important parameters, they are selected using the parameter design. The two-parameter plane of the gain  $K$  and time constant  $\tau_1$  is plotted in Fig. 5. In order to achieve a gain margin of 4 and a phase margin of  $40^\circ$ , the  $K$  and  $\tau_1$  are chosen to be 1.04 and 0.0033.

The result of the parameter design was implemented in an adjustable stiffness and damping controller developed for the magnetic bearing system. The adjustable stiffness and damping controller allows the change in gain and zero in the electric circuit which affect bearing characteristics. The control system design has been proven to be robust by supporting the flywheel without any failure.

## OPTIMAL CONTROL

$H^\infty$  optimal control [5] offers a robust system performance by solving disturbance rejection and plant uncertainties. The  $H^\infty$  control system design minimizes the  $H^\infty$  norm of a pre-designed closed-loop transfer function. Because the  $H^\infty$  norm is the maximum singular value over all frequencies, the controller has good performance even at worst system conditions.

For a stable control system, the  $H^\infty$  norm of the transfer function must be less than 1. The objective of a  $H^\infty$  optimal control is to find a stabilizing controller  $K(s)$  such that the norm is minimized. A mixed sensitivity design is generally used to provide a direct and effective approach for the  $H^\infty$  optimal control. For a mixed sensitivity optimization problem, the objective is to find a stabilizing controller  $K(s)$  such that the norm of the weighted sum of the sensitivity function and the complementary sensitivity function is at a minimum.

The approach employing  $\mu$  synthesis [5] provides a robust control design by solving parameter variations, unstructured uncertainties, and performance requirements. The  $\mu$  synthesis can also be used to analyze robustness of the control system by calculating its structure singular value. The objective of the  $\mu$  synthesis is to find a stabilizing controller and a diagonal scaling matrix such that a minima of a minima of the smallest uncertainty is found. This computation requires iterative processes. The controller is calculated using the  $H^\infty$  control method with a fixed scaling nature which is solved by optimization search techniques with a fixed controller. Theoretically, the  $\mu$  synthesis controller is less conservative compared to the  $H^\infty$  optimal controller but the  $\mu$  synthesis demands more computation time and sometimes does not converge.

Because of the gap growth, the average values for the stiffness increases for the  $H^\infty$  optimal control design. The weighting function  $W_1$  is selected to achieve a bandwidth of at least 250 rad/s and a radial stiffness of 289 N/mm (1650 lb/in). The weighting function is designed to handle the parameter variation caused by the gap growth and unstructured uncertainties at 1000 rad/s. The  $H^\infty$  optimal controller is solved by using computer software MATLAB.

The closed loop frequency response of the FES system at the maximum gap growth is shown in Fig. 6. The system has a flat and smooth response with a bandwidth of 1000 rad/s. The step input response of the system shown in Fig. 7 has an overshoot of 4% and a settling time of 0.003 s.

An optimal controller applying  $\mu$  synthesis was used to handle parameter variations of the stiffness and control energy limitation. The control system has the same physical plant as the  $H^\infty$  optimal controller and a zero order of the diagonal matrix is used to avoid a large order controller design. The control system is designed using MATLAB software. The system has a bandwidth of 70 rad/s at the maximum gap growth as shown in Fig. 8. The step input response of the system shown in Fig. 9 displays an overshoot of 9% and a settling time of 0.009 s.

## CONCLUSION

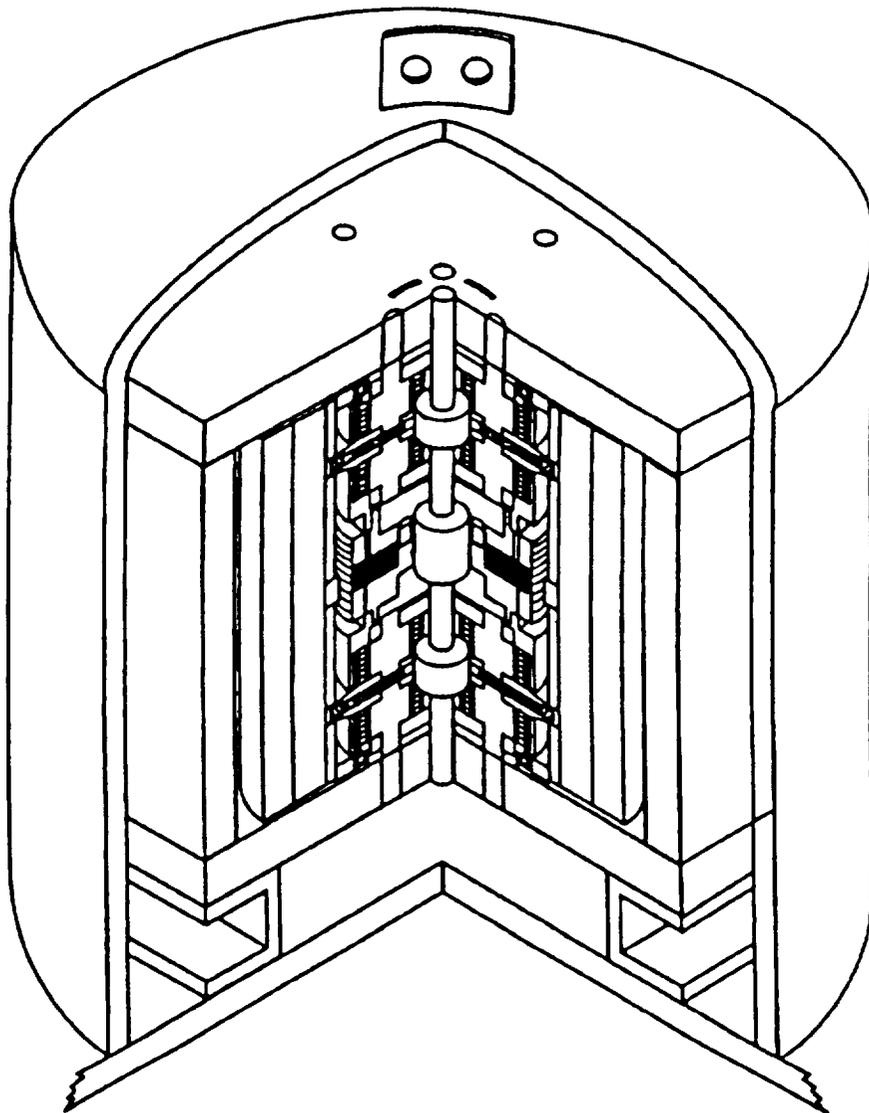
The characteristics and frequency responses of the magnetic bearing system obtained from modeling are comparable to those obtained experimentally. Although magnetic saturation is shown in the bearings, there are good correlations between the theoretical model and experimental data. Both simulation and experiment confirm large variations of the magnetic bearing characteristics due to air gap growth. Therefore, the gap growth effect should be considered in the magnetic bearing system design.

The magnetic bearing control system was designed using three different methods, the parameter design,  $H^\infty$  optimal control, and  $\mu$  synthesis. Because the existing controller using the parameter design never considers any plant uncertainty, it has the worst performance with a limited bandwidth, a large overshoot, and a long settling time. The  $H^\infty$  optimal controller takes into consideration the plant uncertainties, bandwidth, and disturbance attenuation, and achieves the best performance. However, the  $H^\infty$  optimal controller requires very large gain and control energy, which may not be possible in real applications. The optimal controller using  $\mu$  synthesis considers parameter variations of passive stiffness and current force stiffness as well as control energy limitation. It achieves a good performance in bandwidth, settling time, and overshoot. Although its performance is not as good as the  $H^\infty$  optimal controller, it demands less gain and control energy.

## REFERENCES

1. Kirk, J.A.: Flywheel Energy Storage Part I - Basic Concepts. *International Journal of Mechanical Science*, vol. 19, no. 4, 1977, pp. 223-231.
2. Kirk, J.A.; and Studer, P.A.: Flywheel Energy Storage Part II - Magnetically Suspended Superflywheel. *International Journal of Mechanical Science*, vol. 19, no. 4, pp. 233-245.

3. Anand, D.K.; Kirk, J.A.; Zmood, R.B.; Pang, D.; and Lashley, C.: Final Prototype of Magnetically Suspended Flywheel Energy Storage System. *Proceedings of 26th Intersociety Energy Conversion Engineering Conference*, Boston, MA, August 4-11, 1991.
4. Wells, S.; Pang, D.; Kirk, J.A.: Manufacturing and Testing of a Magnetically Suspended Composite Flywheel Energy Storage System. *Proceedings of 2nd International Symposium on Magnetic Suspension Technology*, NASA CP-3247, Seattle, WA, August 11-13, 1993.
5. Pang, D.: Magnetic Bearing System Design for Enhanced Stability. Ph.D. Dissertation, University of Maryland, College Park, MD, 1994.
6. Jayaraman, C.P.: Dynamic Analysis of Magnetic Bearing Stack. M.S. Thesis, University of Maryland, College Park, MD 1988.
7. Zmood, R.B.; Pang, D.; Anand, D.K.; and Kirk, J.A.: On the Nonlinear Operation of Magnetic Bearings. Internal Document, Magnetic Bearing Laboratory, University of Maryland, College Park, MD, 1990.
8. Jeyaseelan, M.: A CAD Approach to Magnetic Bearing Design. M.S. Thesis, University of Maryland, College Park, MD, 1988.
9. Fittro, R.L.: Neural Network Controller Design for a Magnetic Bearing Flywheel Energy Storage System. M.S. Thesis, University of Maryland, College Park, MD, 1993.
10. Chang, C.; and Han, K.: Gain Margins and Phase Margins for Control Systems with Adjustable Parameters. *Journal of Guidance, Control, and Dynamics*, Vol. 13, No. 3, May-June 1990.



**Figure 1 - Schematic of OCCF System**

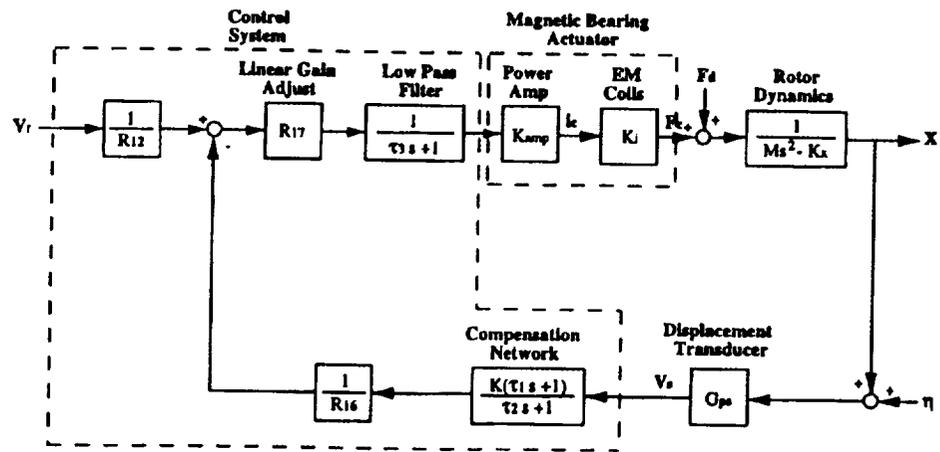


Figure 2 - Linear SISO Model of Magnetic Bearing System

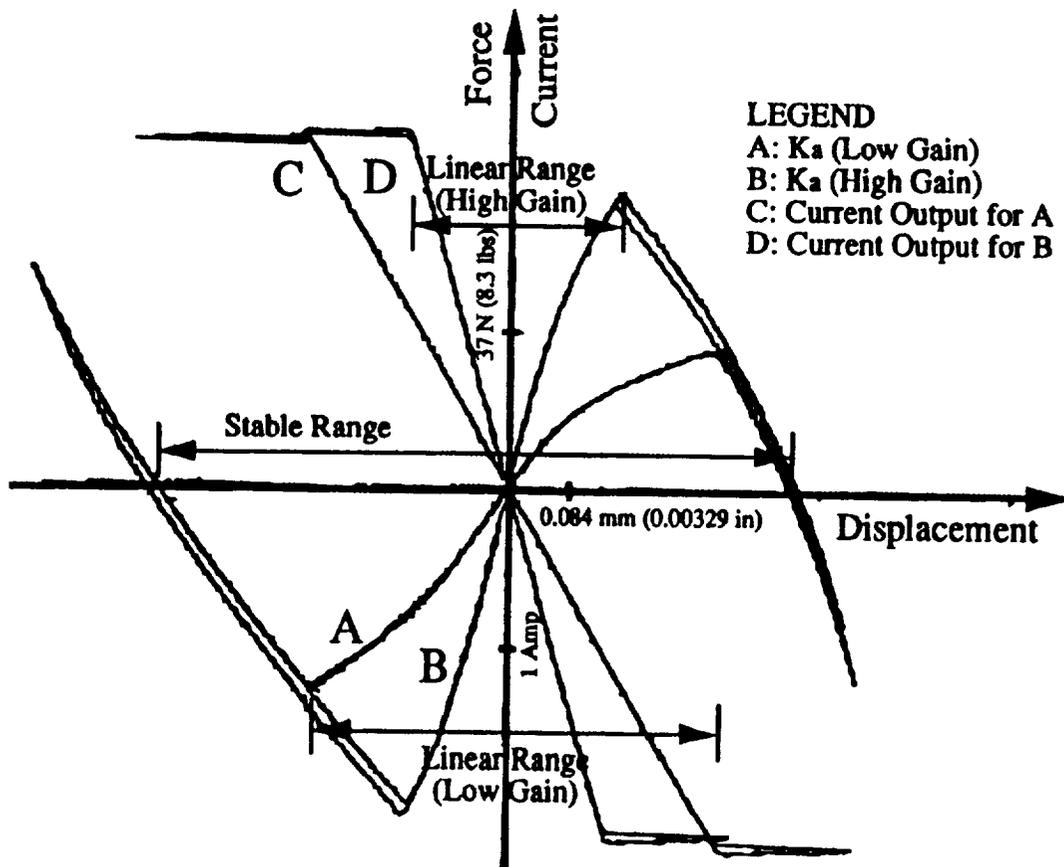


Figure 3 - Experimental Test of PM/EM Magnetic Bearing Characteristics

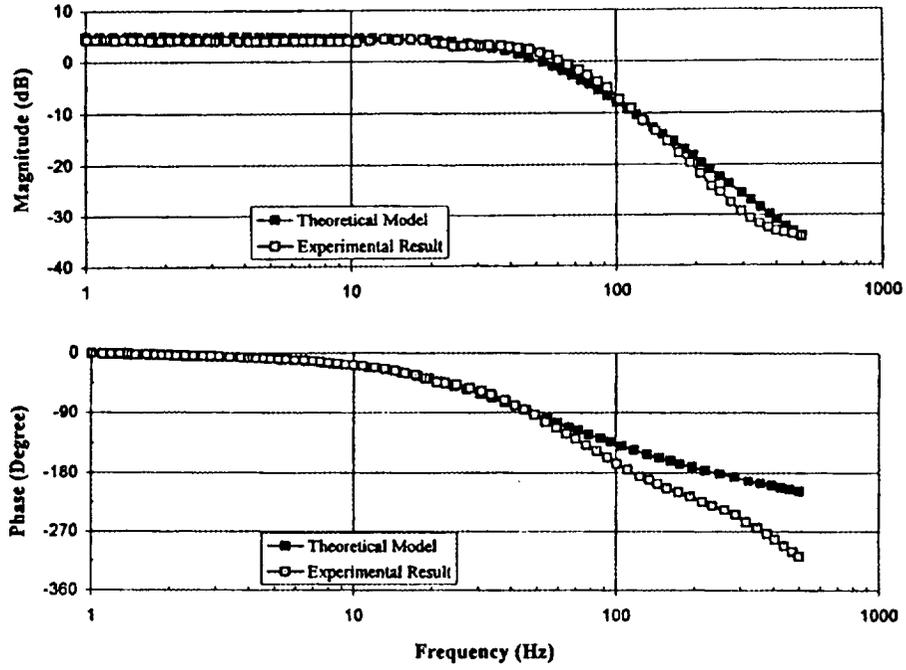


Figure 4 - Displacement Frequency Response of FES System

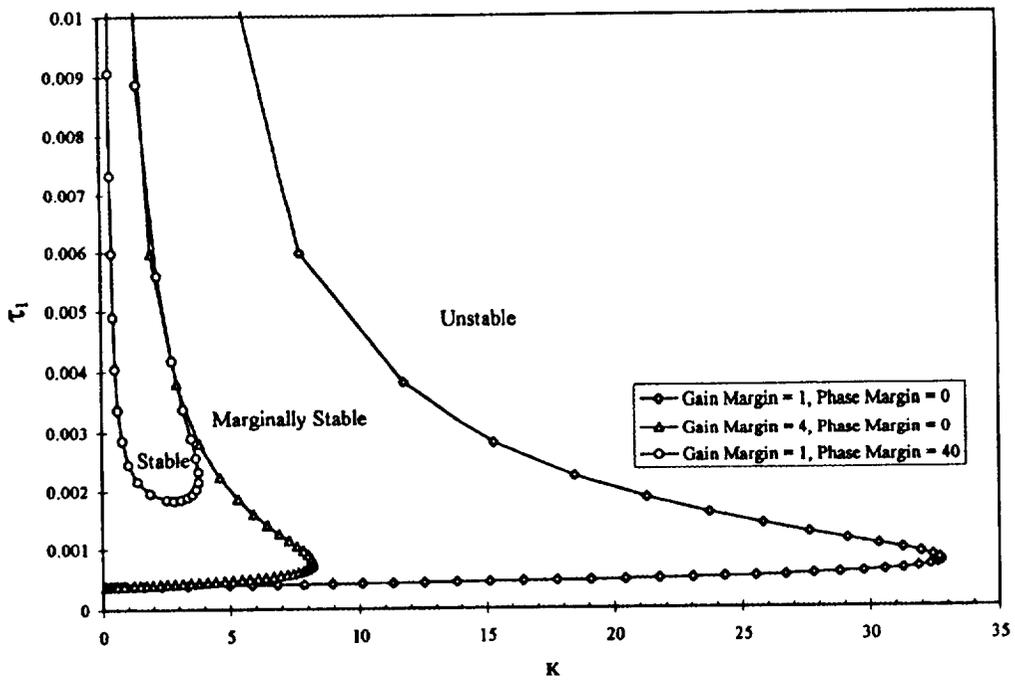
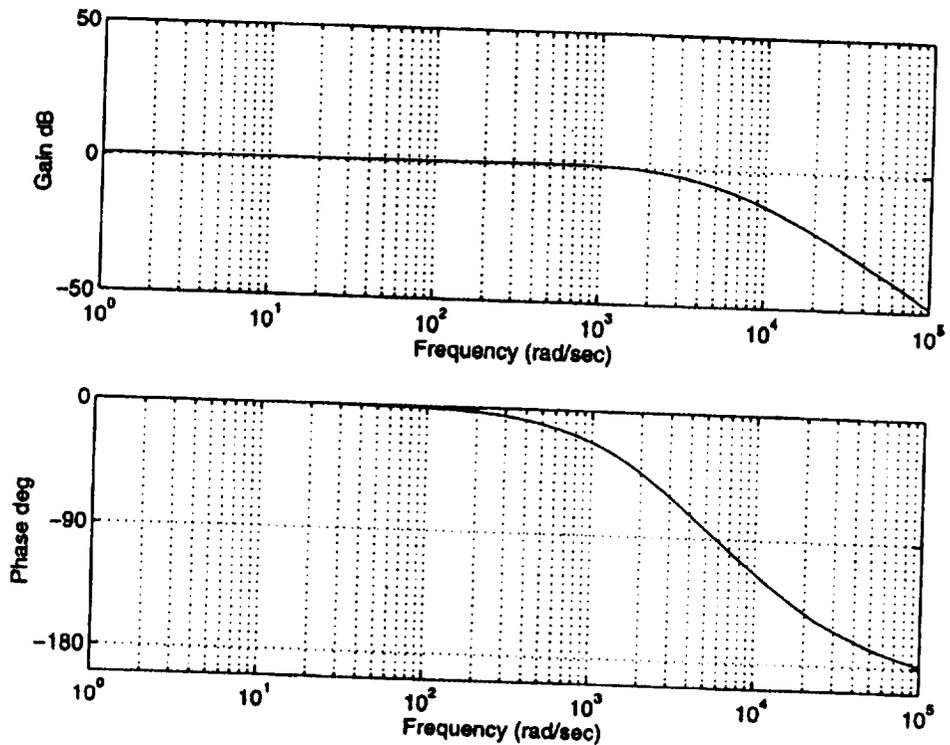
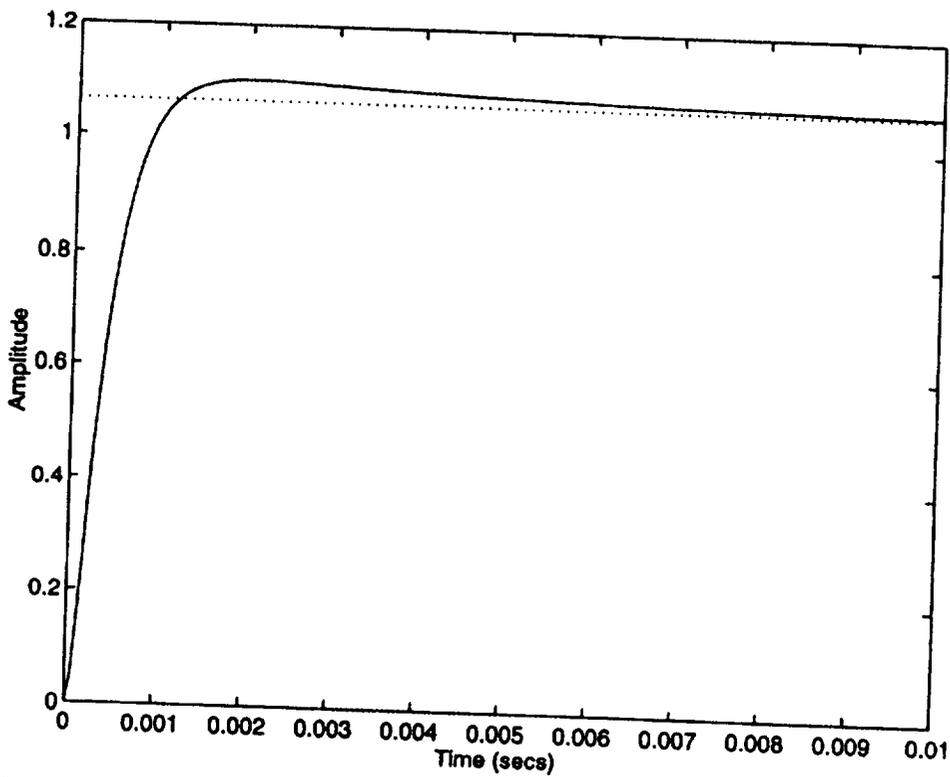


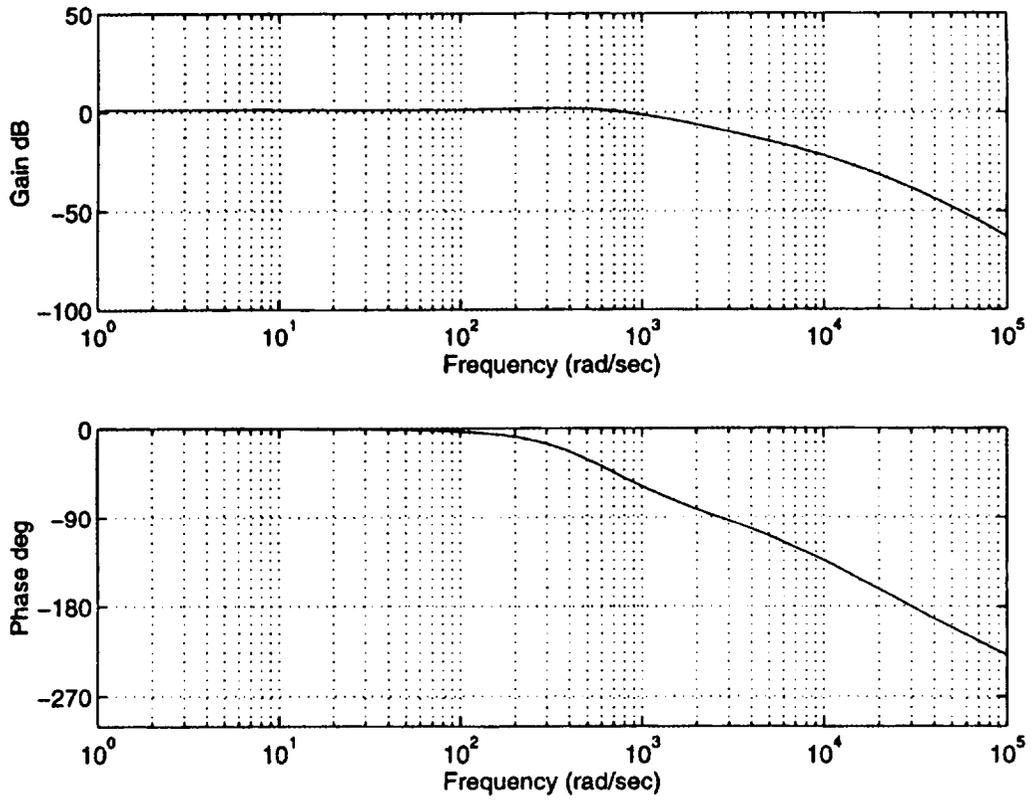
Figure 5 - Control System Parameter Design



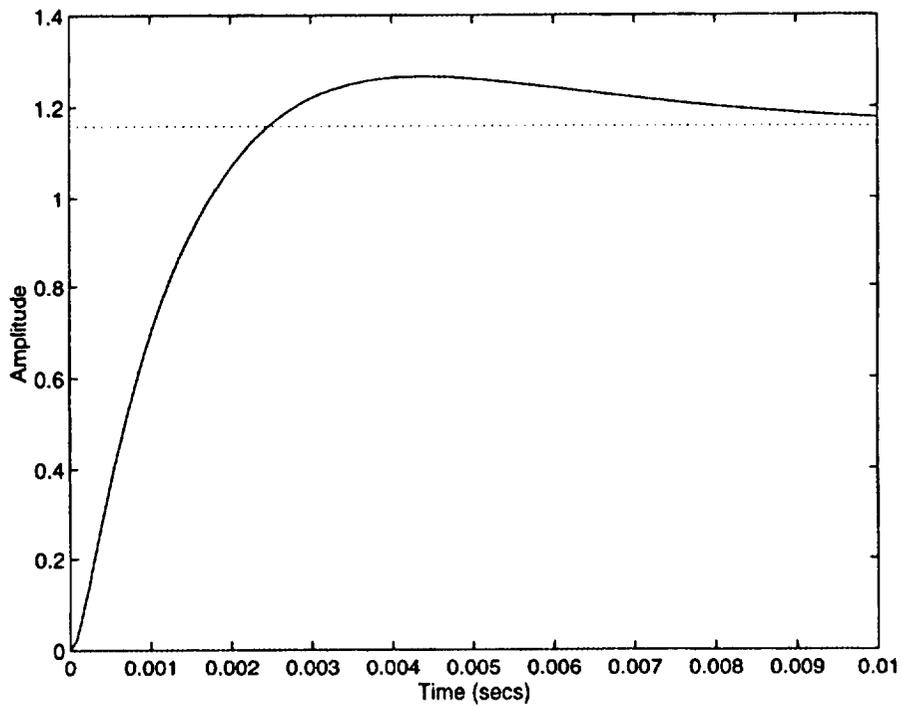
**Figure 6 - Closed Loop Frequency Response Using  $H^\infty$  Optimal Controller**



**Figure 7 - Step Input Response Using  $H^\infty$  Optimal Controller**



**Figure 8 - Closed Loop Frequency Response Using  $\mu$  Synthesis Controller**



**Figure 9 - Step Input Response Using  $\mu$  Synthesis Controller**



**Session 9 -- Controls 2**

Chairman: Carl Knospe  
University of Virginia

